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Higgsless fermion masses and unitarity

Christian Schwinn*

Institut für Physik, Johannes-Gutenberg-Universität,
Staudingerweg 7, D-55099 Mainz, Germany

We discuss the consistency of fermion mass generation by boundary conditions and brane localized terms in higher dimensional Higgsless models of gauge symmetry breaking. The sum rules imposed by tree-level unitarity and Ward Identities are applied to check the consistency of mass generation by orbifold projections and more general boundary conditions consistent with the variational principle. We find that the sum rules are satisfied for boundary conditions corresponding to brane localized mass and kinetic terms consistent with the reduced gauge symmetry on the brane.

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I. INTRODUCTION

Recently a new class of models of electroweak symmetry breaking (EWSB) without Higgs bosons has been proposed [1] in the setting of an additional dimension at the TeV scale [2]. Since Higgsless EWSB is not possible using the abelian orbifold constructions usually employed in higher dimensional models of grand unified theories (GUTs) [3, 4, 5], a more general approach to symmetry breaking by Dirichlet boundary conditions (BCs) has been utilized in these models. This construction has been found to be consistent with the variational principle, tree level unitarity [1] and Becchi-Rouet-Stora-Tyutin (BRST) symmetry [6].

There are by now several variants of this setup in warped space [7, 8, 9] and in flat space [10]. While there is some tension in satisfying precision data and constraints from partial wave unitarity at the same time [8, 10], it has been suggested that these problems can be overcome by appropriate brane kinetic terms (BKTs) for the gauge bosons [9].

In the Higgsless higher dimensional models, the unitarity sum rules (SRs) that guarantee the boundedness of the amplitude at large energies are satisfied because of interlacing cancellations among the Kaluza-Klein (KK)-states of the gauge bosons [1, 11] instead of the exchange of a scalar boson like in 4 dimensional dimensional theories [12, 13]. This observation has also inspired new four dimensional models of Higgsless EWSB [14] with improved unitarity properties. While the most dramatic violations of unitarity can occur in the scattering of massive gauge bosons, in the Standard Model (SM) the Higgs mechanism is also invoked to cancel divergences in the amplitudes for gauge boson production by fermions. Therefore, without a Higgs boson, also an inherently higher dimensional mechanism has to be employed to generate fermion masses without spoiling unitarity. In [15] bulk fermions with BCs corresponding to brane localized mass terms have been proposed for that purpose. Such mass terms have been discussed for Majorana fermions in the context of M-theory or Supergravity breaking [16, 17, 18, 19] or Neutrino masses [20], but brane induced Dirac

*Electronic address: schwinn@thep.physik.uni-mainz.de

masses and the corresponding KK-decomposition in the context of gauge symmetry breaking have been discussed in less detail.

The consistency of the BCs corresponding to boundary mass terms with the variational principle has been discussed in [15] but in gauge theories also the interplay of fermion and gauge boson BCs has to be taken into account in order not to violate unitarity or the Ward identities (WIs) resulting from BRST symmetry. Since gauge symmetry breaking by orbifold and Dirichlet BCs leads to a reduced gauge symmetry on the boundary [4], the inclusion of brane localized terms invariant under the reduced gauge symmetry can be regarded as an explicit but 'soft' symmetry breaking. For brane localized matter fields, the consistency of this explicit breaking with unitarity and WIs has been checked in [4, 6] and in this work we extend this analysis to brane localized mass and kinetic terms of bulk fermions.

In section II we review briefly the main ingredients of Higgsless EWSB and fermion masses. In section III we verify the SRs from tree unitarity and WIs for fermion masses resulting from orbifold projections and discuss the possibility for Higgsless fermion masses in this framework. In section IV, the consistency of fermion masses from brane localized mass and kinetic terms and from mixing with brane fermions is discussed.

II. HIGGSLESS EWSB AND FERMION MASSES

We now outline the mechanism of Higgsless EWSB by Dirichlet BCs and the generation of fermion masses by boundary terms in the flat space toy model of [1, 15]. This is not yet a viable model and to obtain the correct masses for the weak gauge bosons, one must either introduce a warped compactification [7] or include brane localized kinetic terms [10]. However for the issue of fermion mass generation, the essential features can be discussed already in the simpler framework provided by the flat space toy model.

A. Higgsless gauge symmetry breaking

The mechanism of Higgsless EWSB proposed in [1] prescribes Dirichlet BCs to the gauge bosons associated to the broken symmetry generators (identified by a hat) and Neumann BCs to the unbroken gauge bosons:

$$A_\mu^{\hat{a}}(y_f) = 0 \quad , \quad \partial_y A_\mu^a(y_f) = 0 \tag{1}$$

where y_f denotes one of the endpoints of the interval $[0, \pi R]$ of the fifth dimension. This symmetry breaking by BCs allows to avoid the group theoretical constraints from abelian orbifold symmetry breaking [5]. Physically, the Dirichlet BCs can arise from the coupling to a boundary Higgs boson with a vacuum expectation value that is pushed to infinity [1, 5]. The important common property of Dirichlet BCs and orbifold breaking ensuring unitarity of gauge boson scattering [1] and BRST symmetry [6] is the vanishing of the wavefunctions of the broken gauge bosons at the branes. Via the gauge transformation law, this implies also the vanishing of the gauge parameters corresponding to the broken generators (and the vanishing of the wavefunctions of the ghosts associated to the broken gauge bosons in the BRST formalism [6]). Therefore the concept of a reduced gauge or BRST symmetry on the branes can be introduced similar to the orbifold case [4].

The group structure of Higgsless EWSB employed in [1] is a left-right symmetric bulk symmetry group $SU(2)_L \times SU(2)_R \times U(1)$. On the brane at $y = \pi R \equiv \ell$ the left-right sym-

metry is broken to the diagonal subgroup $SU(2)_{L+R}$ by assigning Dirichlet BCs to the broken components $A_\mu^{-,a} \equiv \frac{1}{\sqrt{2}}(A_\mu^{L,a} - A_\mu^{R,a})$ while the unbroken components $A_\mu^{+,a} \equiv \frac{1}{\sqrt{2}}(A_\mu^{L,a} + A_\mu^{R,a})$ and B_μ satisfy Neumann BCs. Analogously, on the second brane at $y = 0$ the symmetry is broken according to $SU(2)_R \times U(1) \to U(1)_Y$ by prescribing Dirichlet BCs to the fields $A_\mu^{R,1/2}$ and $(g'B_\mu - gA_\mu^{R,3})$ while the unbroken fields are $A_\mu^{L,a}$ and the linear combination $(gB_\mu + g'A_\mu^{R,3})$. The only unbroken component that satisfies Neumann BCs at both branes and therefore possesses a massless zero mode is the combination $\gamma_\mu := gB_\mu + g'(A_\mu^{R,3} + A_\mu^{L,3})$.

Using a theory space approach, a simpler setup involving only a bulk $SU(2)$ has been proposed [14] but a prescription how to incorporate fermions in a true five dimensional version of the model has not been given.

B. Higgsless fermion masses

To reproduce the SM fermion spectrum, in [15] a bulk fermion is introduced for every chiral fermion. As an example, the left handed doublet (u_L, d_L) is the zero mode of a bulk $SU(2)_L$ doublet while the righthanded quarks u_R, d_R are contained in a bulk $SU(2)_R$ doublet[1]

$$\begin{aligned}
\Psi_{Q_L} &= (\Psi_{u_L}, \Psi_{d_L}) = \left(\begin{pmatrix} u_L \\ \eta_{u_L} \end{pmatrix}, \begin{pmatrix} d_L \\ \eta_{d_L} \end{pmatrix} \right) \\
\Psi_{Q_R} &= (\Psi_{u_R}, \Psi_{d_R}) = \left(\begin{pmatrix} \chi_{u_R} \\ u_R \end{pmatrix}, \begin{pmatrix} \chi_{d_R} \\ d_R \end{pmatrix} \right)
\end{aligned} \tag{2}$$

The righthanded massless modes of Ψ_{Q_L} and the lefthanded massless modes of Ψ_{Q_R} can be projected out by BCs similar to those of fermions on an orbifold [21, 22] (see section III A).

In [15], brane localized mass terms are used to generate Dirac masses for the zero modes of bulk fermions. On the brane at $y = \ell$, only the diagonal subgroup of $SU(2)_L \times SU(2)_R$ is unbroken and Dirac masses terms consistent with this symmetry can be added:

$$\mathscr{L}_\ell = -\delta(y - \ell) M_D \ell (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) \tag{3}$$

On the brane at $y = 0$, the broken $SU(2)_R$ allows to lift the degeneracy of the masses in the isospin multiplets. A mass splitting in the quark sector can be generated by introducing brane localized vectorlike fermions $\psi_u = (\chi_u, \eta_u)$ that have the same quantum numbers as the up-type quarks. One can then add a brane term mixing the right handed bulk up-type quarks with the brane fermions

$$\mathscr{L}_0 = -\delta(y) M \ell^{1/2} (\bar{u}_R \chi_u + \bar{\chi}_u u_R) \tag{4}$$

to generate a mass splitting among the up-and down type quarks. The mass splitting in the lepton doublet can be generated analogously. As noted in [15], a similar mass spectrum can be obtained from BKTs [23] for the right handed bulk fermions without the need to introduce brane localized fermions.

As in the model of Nomura in [7] there are other less minimal possibilities to obtain the SM fermion spectrum, employing the same ingredients of brane mass terms and mixing with brane fermions. In the remainder of this work, we consider generic features of BKTs and

[1] Our notation for fermions in five dimensions is introduced in section III A and appendix A 2

mass terms of the form (3) and (4) so our conclusions apply to any construction using these ingredients. This should also be useful in a more general context than EWSB like Higgsless breaking of GUT symmetries that is not possible from orbifolding, e.g. the breaking of $SO(10)$ to the SM in five dimensions.

III. CONSISTENCY OF FERMION MASSES FROM ORBIFOLDING

Since the symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ used in Higgsless EWSB is isomorphic to the allowed orbifold symmetry breaking pattern [5] $SO(4) \to SO(3)$, it can also be described in terms of orbifold breaking, as already noted in [1]. In section III A we discuss the generation of fermion masses from this perspective, providing an alternative to the brane localized mass terms. We then apply the methods of [6] to bulk fermions and verify the consistency of gauge symmetry breaking by orbifold BCs. The extension to the more general BCs resulting from brane localized terms is discussed in section IV.

A. Orbifold symmetry breaking and Higgsless fermion masses

Before we turn to the orbifold description of the symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ we need to recall some results on fermions on a 5 dimensional orbifold [21, 22] in the connection of gauge symmetry breaking (see e.g. [5, 24]). The orbifold $S/(Z_2 \times Z_2')$ is obtained from the circle by identifying points under the action of the two reflections $Z_2 : y \to -y$ and $Z_2' : (y - \pi R) \to -(y - \pi R)$. Fields defined on an orbifold need only be invariant under the reflections up to transformations $\mathcal{Z}$ that are a representation of Z_2 in field space and leave the lagrangian invariant. In the following, gauge bosons in five dimensions are decomposed into a four dimensional vector and a scalar according to[2] $A_M^a = (A_\mu^a, -\phi^a)$. For the case of a five dimensional gauge theory coupled to fermions, the combined orbifold transformations take the form [24]

$$\begin{aligned} A_\mu^a(x, -(y - y_f))) &= \mathcal{Z}_{ab}^{y_f} A_\mu^b(x, y - y_f) \\ \phi^a(x, -(y - y_f)) &= -\mathcal{Z}_{ab}^{y_f} \phi^b(x, y - y_f) \\ \Psi_i(x, -(y - y_f)) &= \gamma^5 \lambda_{ij}^{y_f} \Psi_j(x, y - y_f) \end{aligned} \tag{5}$$

with $y_f = 0, \ell = \pi R$. The representation matrices $\mathcal{Z}_{ab}^{y_f}$ must satisfy $\mathcal{Z}^2 = 1$ so their eigenvalues are $\eta_a^{y_f} = \pm 1$. For $\eta_a^{y_f} \neq 1$, the wavefunctions of the vector components of the gauge fields must vanish at the fixed point y_f so there the gauge symmetry is broken to a subgroup H. The λ are hermitian matrices acting in the representation of the fermions, satisfying in addition $\lambda = \lambda^{-1}$. In order to leave the interaction with the gauge bosons invariant under the transformation (5), in the basis where the $\mathcal{Z}$ are diagonal, the condition

$$\lambda \tau^a \lambda = \eta_a \tau^a \tag{6}$$

must be satisfied. Here the τ are the generators of the gauge group in the representation of the fermions.

[2] See appendix A for our conventions for the KK decompositions and the resulting effective lagrangian of the KK-modes.

The KK-decomposition of five dimensional fermions is introduced as (c.f. appendix A 2 for details)

$$\Psi_i(x,y) = \sum_n \begin{pmatrix} \chi_{i,n}(x)\zeta^-_{i,n}(y) \\ \eta_{i,n}(x)\zeta^+_{i,n}(y) \end{pmatrix} \tag{7}$$

where the 4-dimensional spinors of the KK modes satisfy the Dirac equation (A.14).

In the basis where the λ^{y_f} are diagonal with eigenvalues $\lambda_i^{y_f} = \pm 1$, the KK modes transform under the orbifold transformation according to [21, 22]

$$\zeta^\pm_{i,n}(-(y-y_f)) = \pm\lambda_i^{y_f}\zeta^\pm_{i,n}(y-y_f) \tag{8}$$

Therefore for $\lambda_i = 1$ only the right-handed fermions possess zero-modes, for $\lambda_i = -1$ only the left-handed.

We will now introduce an orbifold approach to the breaking $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ and discuss how Dirac masses for the lightest fermions can be obtained. We collect the gauge fields in a vector $A_M = (A_L, A_R)$ and assign the orbifold parities

$$\mathscr{Z}^0 = \mathbf{1} \quad , \quad \mathscr{Z}^\ell = \sigma^1 \tag{9}$$

where the sigma matrices act on the L/R indices, not on the indices of the gauge group. The transformation (9) is diagonalized by going to the basis

$$A_M^{\pm,a} = \frac{1}{\sqrt{2}}(A_M^{L,a} \pm A_M^{R,a}) \tag{10}$$

so that $\eta^\ell_\pm = \pm 1$.

As a simple model for the setup discussed in section II, we will consider two bulk fermion doublets: Ψ_L charged under $SU(2)_L$ and Ψ_R charged under $SU(2)_R$. Assembling Ψ_L and Ψ_R in a vector $\Psi = (\Psi_L, \Psi_R)$, the interaction lagrangian of the fermions can be written as

$$\mathscr{L}_{f,\mathrm{int}} = \frac{1}{\sqrt{2}}\bar\Psi_i(x,y)\tau^a_{ij}\Gamma^M\left[A_M^{+,a}(x,y) + \sigma_3 A_M^{-,a}(x,y)\right]\Psi_j(x,y) \tag{11}$$

In contrast to the example of five dimensional QED [22], the left-right symmetric gauge symmetries in the bulk forbid the addition of explicit bulk mass terms connecting Ψ_R and Ψ_L. Instead, we can give mass to the lightest KK modes using an appropriate orbifold transformation. The generators of the transformation of the fermions must satisfy (6), i.e. λ^ℓ has to anticommute with σ^3. To mix Ψ_L and Ψ_R we choose

$$\lambda^0 = -\sigma_3 \quad , \quad \lambda^\ell = \sigma_1 \tag{12}$$

where again the sigma matrices act on the L/R indices. More explicitly, the transformations of the fermions are given by

$$\begin{aligned} \Psi_{L/R}(x,-y) &= \mp\gamma^5\Psi_{L/R}(x,y) \\ \Psi_L(x,-(y-\ell) &= \gamma^5\Psi_R(x,y-\ell) \end{aligned} \tag{13}$$

so at $y = 0$ we project out the righthanded zero modes of Ψ_L and the lefthanded zero modes of Ψ_R while the transformation at $y = \ell$ mixes Ψ_L and Ψ_R in order to generate Dirac masses for the surviving zero modes. The orbifold symmetries (13) are consistent with a bulk mass term $m(y)(\bar\Psi\Psi)$ where $m(y)$ is an odd function under the orbifold transformations.

As a simple example, we consider a mass term given by a step function $m(y) = m\epsilon(y)$. The KK-wavefunctions for more usual BCs have been given e.g. in [15]. Imposing the orbifold condition (13) at $y = \ell$ we obtain the KK wavefunctions in the interval $[0;\ell]$ up to normalization as

$$\begin{aligned} \zeta^-_{L,n}(y) &= \left(\cos(k_n y) + \tfrac{m}{k_n}\sin(k_n y)\right) \\ \zeta^+_{L,n}(y) &= \tfrac{m_n}{k_n}\sin(k_n y) \\ \zeta^-_{R,n}(y) &= -\beta_n \tfrac{m_n}{k_n}\sin(k_n y) \\ \zeta^+_{R,n}(y) &= \beta_n \left(\cos(k_n y) - \tfrac{m}{k_n}\sin(k_n y)\right) \end{aligned} \tag{14}$$

with $\beta_n m_n = m \pm \sqrt{m^2 + m_n^2}$ and $k_n^2 = m_n^2 - m^2$. The quantization condition for the masses is given by:

$$\begin{aligned} \zeta^-_{L,n}(\pi R) &= -\zeta^-_{R,n}(\pi R) \\ \Leftrightarrow \quad \tan(k_n \pi R) &= \pm \frac{k_n}{\sqrt{m^2 + m_n^2}} \end{aligned} \tag{15}$$

Therefore, our construction indeed gives Dirac masses connecting the left-handed component of Ψ_L and the right handed component of Ψ_R that are determined by the bulk mass m. It is beyond the scope of this paper to discuss the viability of this setup in the context of a realistic model. At any rate, the split of the masses within the isospin multiplets has to occur in the context of the $SU(2)_R \times U(1) \to U(1)_Y$ breaking at $y = 0$ that cannot be achieved by orbifolding alone.

B. Consistency with unitarity and WIs

Having described how to obtain fermion masses from orbifold BCs, we now turn to the verification of the consistency of orbifold symmetry breaking, using tree level unitarity and WIs as criteria. In KK-gauge theories, using an appropriate gauge fixing, WIs similar to those in a four dimensional spontaneously broken gauge theory (SBGT) are valid with the scalar component of the five dimensional gauge boson taking the role of the Goldstone bosons (GBs) [6, 11]. In [6] it has been shown that the SRs ensuring unitarity cancellations can also be derived by imposing those WIs on a minimal set of scattering amplitudes. Thus the unitarity conditions for the fermion couplings [12] can be obtained in a simpler way from the WIs for the $\bar{f}f \to WW$ amplitude.

In gauge boson production from fermions, the cancellation of the terms growing with the square of the energy is ensured by the relation (see (A.9) and (A.17) for the definition of the coupling constants)

$$\mathcal{T}^\alpha_{R/LIJ}\mathcal{T}^\beta_{R/LJK} - \mathcal{T}^\beta_{R/LIJ}\mathcal{T}^\alpha_{R/LJK} = g^{\alpha\beta\gamma}\mathcal{T}^\gamma_{R/LIK} \tag{16a}$$

that has the form of a Lie algebra for the $\mathcal{T}_{R/L}$. Here we have combined the KK and the group indices into multi-indices $(n,i) \equiv I$ and $(a,i) \equiv \alpha$ and have used a summation convention. The cancellation of the subleading divergences $\propto E$ implies the relation

$$\mathrm{i}g^\beta_{LIJ}\mathcal{T}^\alpha_{LJK} - \mathrm{i}\mathcal{T}^\alpha_{RIJ}g^\beta_{LJK} = g^\gamma_{LIK}T^\alpha_{\beta\gamma} \tag{16b}$$

This relation can be interpreted as an invariance condition of the Yukawa coupling g^β_{LIJ} under transformations generated by the T and $\mathcal{T}$. In general, an additional term involving

the Higgs coupling appears in this condition [6, 12]. In (16b) the coupling of the fermions to the GBs g^{γ}_{LIK} has to satisfy the relation

$$m_\alpha g^{\alpha}_{LIJ} = -\mathrm{i}(m_I \mathcal{T}^{\alpha}_{LIJ} - m_J \mathcal{T}^{\alpha}_{RIJ}) \tag{17a}$$

and the coupling of the GBs to the gauge bosons has to satisfy

$$T^{\alpha}_{\beta\gamma} = \frac{1}{2m_\beta m_\gamma} g^{\alpha\beta\gamma}(m_\alpha^2 - m_\beta^2 - m_\gamma^2) \tag{17b}$$

These conditions arise from the WIs for three point vertices [6].

The fulfillment of the SRs (16) and (17) is a necessary but no sufficient condition for perturbative unitarity. In four dimensional SBGTs, an upper bound on the Higgs mass can be obtained demanding that unitarity cancellations set in before partial wave unitarity gets violated [13]. Although a rigorous derivation has not yet been given, similar considerations in Higgsless higher dimensional models lead to an upper bound on the masses of the lightest KK excitations of the gauge bosons as discussed by Davoudiasl *et.al* in [8]. Furthermore, partial wave unitarity in compactified higher dimensional gauge theories is violated by the infinite number of KK modes so these theories have to be considered as effective theories valid below a scale determined by the higher dimensional dimensionfull gauge coupling constant [11]. In the following, we will only be concerned with the SRs (16) and (17) ensuring the cancellation of the terms diverging with the energy.

To check (17a), using the relation among the gauge boson wavefunctions (A.7) and the equations of motion (A.15) we obtain, integrating by parts:

$$\begin{aligned} m_\alpha g^{\alpha}_{LIJ} &= \mathrm{i}\tau^a_{ij} \int \mathrm{d}y\, \zeta^+_I(y)\zeta^-_J(y)\partial_y f^\alpha(y) \\ &= -\mathrm{i}(m_I \mathcal{T}^{\alpha}_{LIJ} - m_J \mathcal{T}^{\alpha}_{RIJ}) + \mathrm{i}\tau^a_{ij}[\zeta^+_I \zeta^-_J f^\alpha]_{y_f} \end{aligned} \tag{18}$$

This is in agreement with the result (17a) from the WI provided the boundary term vanishes. For the coupling to broken gauge bosons this follows since the gauge boson wavefunction f^a at the fixed point vanishes for orbifold or Dirichlet BCs. Then (17a) is satisfied independent of the BCs of the fermions.

For the coupling to unbroken gauge bosons we have to use the consistency condition (6). In the basis where the orbifold transformation of the gauge bosons is diagonal it implies

$$\tau^a_{ij}(\lambda_i - \lambda_j) = 0 \tag{19}$$

and therefore $\lambda_i = \lambda_j$ if the generators are nonvanishing. Thus either ζ^+ or ζ^- vanishes since the left and right handed modes have different orbifold parity. Therefore (17a) is satisfied for orbifold gauge symmetry breaking. The consistency condition for a more general symmetry breaking by BCs is that the couplings of the unbroken gauge bosons must connect fermions with the same BCs.

The relation (17b) can be verified analogously using the relation among the wavefunctions (A.7), integrating by parts two times and using the equation of motion for the KK-

wavefunctions (A.4):

$$
\begin{aligned}
m_{\alpha_y} m_{\beta_y} T^{\alpha}_{\beta\gamma} &= f^{abc} \int \mathrm{d}^N y \, f^{\alpha}(y) \partial_y f^{\beta}(y) \partial_y f^{\gamma}(y) \\
&= \frac{1}{2} f^{abc} \int d^N y \left[\partial_y^2 f^{\alpha}(y) f^{\beta}(y) f_l^{\gamma}(y) - f^{\alpha}(y) \partial_y^2 f^{\beta}(y) f^{\gamma}(y) - f^{\alpha}(y) f^{\beta}(y) \partial_y^2 f^{\gamma}(y) \right] \\
&= \frac{1}{2} (m_\alpha^2 - m_\beta^2 - m_\gamma^2) g^{\alpha\beta\gamma}
\end{aligned}
\tag{20}
$$

The boundary terms occurring in the integration by parts are of the form $[\partial_y f^a f^b f^c]$. As has been shown in [6] these terms vanish as long as the wavefunctions of the broken gauge bosons are zero on the boundary so both orbifold and general Dirichlet BCs (1) are consistent with unitarity and WIs.

Conditions similar to (16) have been discussed in detail for the gauge boson SRs in [1, 6] so here we will be brief. Performing the sum over the KK-modes using the completeness relations for the fermion and gauge boson wavefunctions, the same integral over the KK wavefunctions appears in every term and both equations of (16) reduce to the Lie algebra of the generators of the gauge group. For instance, the condition (16b) turns into

$$
0 = \left([\tau^a, \tau^b] - \mathrm{i} f^{abc} \tau^c \right) \int dy \; f^{\alpha}(y) g^{\beta}(y) \zeta_I^{+}(y) \zeta_J^{-}(y) \tag{21}
$$

Note that the unitarity cancellations require to sum over the KK-towers of both fermions and gauge bosons so it is essential that the fermions propagate in the bulk.

IV. CONSISTENCY OF BRANE LOCALIZED TERMS

We now turn to the verification of the SRs for theories including brane localized terms like (3) and (4). In section IV A we review the consistency of BCs with the equations of motion. In section IV B we discuss the consistency of the BCs corresponding to brane localized masses, extending the analysis of section III B. Brane localized kinetic terms and mixing with brane fermions are discussed in section IV C.

A. Generalized boundary conditions

There are two approaches to theories on an orbifold: in the interval approach (sometimes called ‘downstairs’ approach) one uses continuous fields on the physical interval $[0, \pi R]$ while in the orbifold (or ‘upstairs’) approach fields are defined on the circle $[0, 2\pi R[$ and discontinuities are allowed at the orbifold fixed points $0, \pi R$. The treatment of brane localized terms like (3) and (4) differs in the two approaches. In the interval approach, one imposes appropriate BCs at the boundaries instead of including the singular terms involving delta functions in the equations of motion [15]. In the orbifold approach, singular terms are included in the equations of motion and ordinary orbifold BCs are imposed at the fixed points. Nontrivial BCs that determine the discontinuities in the wave functions and the mass spectrum are derived by integrating the equations of motion in an infinitesimal interval around the fixed points [18, 19].

In this section we take the interval point of view and review possible BCs consistent with the equations of motion, following [15] but focusing on models with a left-right symmetry in the bulk and BCs corresponding to Dirac masses. (For a discussion of consistent BCs for fermions in six dimensions see [25]).

In order to obtain equations of motion without boundary terms, the BCs have to be chosen so that the boundary terms in the variational derivation of the equations of motion vanish. To obtain these conditions, we write the kinetic term in the symmetric form

$$\mathscr{L}_{\mathrm{kin}} = \frac{1}{2}\mathrm{i}(\bar{\Psi}\partial_M\Gamma^M\Psi - \partial_M\bar{\Psi}\Gamma^M\Psi) \tag{22}$$

where again $\Psi = (\Psi_L, \Psi_R)$. The boundary terms appearing in the variation of the action are given by

$$\frac{1}{2}\int \mathrm{d}^4x \left[(\delta\bar{\Psi})\gamma^5\Psi - \bar{\Psi}\gamma^5\delta\Psi\right]_0^\ell = \frac{1}{2}\int \mathrm{d}^4x \left[\delta\chi^\dagger\eta - \delta\eta^\dagger\chi - \chi^\dagger\delta\eta + \eta^\dagger\delta\chi\right]_0^\ell \tag{23}$$

We will now impose the BC that the term in brackets vanishes at each boundary $y_f = 0, \ell$.

Of course the simplest solution is to demand that every term in (23) vanishes by itself, e.g. by demanding

$$\eta_L(y_f) = \chi_R(y_f) = 0 \tag{24}$$

Here also the corresponding variations are demanded to vanish. This corresponds just to the orbifold BCs discussed in the previous section. A less trivial solution corresponds to the introduction of a brane Dirac mass term (c.f. appendix B 1). In contrast to the case of Majorana fermions discussed in detail in [15], we cannot demand BCs of the form $\eta_{L/R} \propto \chi_{L/R}$. We can, however, choose a BC that mixes Ψ_L and Ψ_R:

$$\begin{aligned} \chi_L(y_f) &= -\tan\alpha_{y_f}\chi_R(y_f) \\ \eta_L(y_f) &= \cot\alpha_{y_f}\eta_R(y_f) \end{aligned} \tag{25}$$

so that

$$\left[\delta\chi_L^\dagger\eta_L + \delta\chi_R^\dagger\eta_R\right]_{y_f} = 0 \tag{26}$$

and so on. This generalizes the orbifold BC (13) that corresponds to the special case $\alpha = \frac{\pi}{4}$. Instead of (15) we obtain the mass quantization condition

$$\frac{\zeta^-_{L,n}(y_f)}{\zeta^-_{R,n}(y_f)} = -\tan\alpha_{y_f} \tag{27}$$

Another choice of BCs consistent with the variational principle is [15]

$$\chi(y_f) = \mathrm{i}\kappa\sigma^\mu\partial_\mu\eta(y_f) \tag{28}$$

This BC corresponds to a brane kinetic term (c.f. appendix B 2). Here the boundary terms (23) vanish since the operator $\mathrm{i}\sigma^\mu\partial_\mu$ is hermitian:

$$\int \mathrm{d}^4x \left[\delta\chi_R^\dagger\eta_R - \delta\eta_R^\dagger\chi_R\right]_{y_f} = -\mathrm{i}\kappa\int \mathrm{d}^4x \left[(\partial_\mu\delta\eta_R^\dagger\sigma^\mu)\eta_R + \delta\eta_R^\dagger(\sigma^\mu\partial_\mu\eta_R)\right]_{y_f} = 0 \tag{29}$$

A generalization of (28) appears for mixing with brane localized fermions (c.f. section IV C).

The advantage of the interval approach adopted in this section is that discontinuous wavefunctions and the associated ambiguities are avoided. In gauge theories, however, consistency with the variational principle is not the only consistency requirement. Assigning BCs at will can violate unitarity or WIs, even if the action is gauge invariant and the BCs are consistent with the equations of motion. A drawback of the interval approach is that the compatibility of the BCs with the gauge symmetry is not apparent while these issues are much more transparent in the equivalent description in terms of brane localized terms on orbifolds. Therefore both approaches will be taken into account in the subsequent discussion of the consistency of BCs, providing useful cross checks of the results.

As a further alternative to brane located mass terms, an equivalent description in terms of Scherk-Schwarz breaking on orbifolds has been found for suitable Majorana brane mass terms [17]. It would be interesting to extend this analysis to brane induced Dirac masses in the context of gauge symmetry breaking, but this is beyond the scope of this work.

B. Brane localized Dirac masses

We now extend the discussion of section III B to the more general BCs (25). As discussed in appendix B 1 such BCs arise also from a brane localized mass term. We consider the same gauge symmetry breaking pattern $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ like in section III A but rather than mixing Ψ_L and Ψ_R by an orbifold transformation as in (13), this is achieved by the mass term (3). While in a realistic model this must be combined with isospin breaking brane terms at $y=0$, only the BCs at $y=\ell$ are important in the subsequent discussion and the consistency of the brane localized terms at the other boundary can be discussed separately (see section IV C).

For simplicity, we set the bulk masses to zero so the KK wavefunctions of the left-and right handed wavefunctions are related as

$$\begin{aligned} \zeta^-_{L,I}(y) &= \zeta^+_{R,I}(y) \equiv \zeta^-_I(y) \\ \zeta^+_{L,I}(y) &= -\zeta^-_{R,I}(y) \equiv \zeta^+_I(y) \end{aligned} \tag{30}$$

(see (14) for $m=0$). The equations of motion (B.1) imply the equations for the KK-wavefunctions:

$$\partial_5 \zeta^\pm_I \mp m_I \zeta^\mp_I + \delta(y-\ell) M_D \ell\, \zeta^\mp_I = 0 \tag{31}$$

As derived in appendix B 1, the KK wavefunctions satisfy the BCs (27) with $\alpha = \mathrm{artanh} M_D \ell$ (see (B.4)). This agrees with the results of [19] for Majorana mass terms.

Performing the KK-decomposition of the Lagrangian (11), we obtain the interaction terms

$$\mathscr{L}_{f,KK} = \frac{1}{\sqrt{2}} \bar\psi_I (\not{A}^+_\alpha \mathcal{T}^{+\alpha}_{IJ} + \not{A}^-_\alpha \mathcal{T}^{-\alpha}_{IJ} \gamma^5) \psi_J + \bar\psi_I (\phi^+_\alpha g^{+\alpha}_{IJ} + \phi^-_\alpha g^{-\alpha}_{IJ} \gamma^5) \psi_J \tag{32}$$

with the coupling constants given by

$$\mathcal{T}^{\pm\alpha}_{IJ} = \tau^a_{ij} \int \mathrm{d}y \left[\zeta^+_I(y) \zeta^+_J(y) \pm \zeta^-_I(y) \zeta^-_J(y) \right] f^{\alpha,\pm}(y) \tag{33a}$$

$$g^{\pm\alpha}_{IJ} = \mp \mathrm{i} \tau^a_{ij} \int \mathrm{d}y \left[\zeta^+_I(y) \zeta^-_J(y) \mp \zeta^-_I(y) \zeta^+_J(y) \right] g^{\alpha,\pm}(y) \tag{33b}$$

Following the discussion of the pure orbifold symmetry breaking in section III B, we now verify the unitarity SRs. The SRs (16) are satisfied like in the pure orbifold case by the completeness relation of the KK-wavefunctions.

To check the condition (17a), let us first employ the interval approach where we use the equations of motion without delta-singularities and impose the nontrivial BCs (27) instead. Similarly to (18) we obtain

$$m_\alpha g_{IJ}^{\pm\alpha} = \pm \mathrm{i}\tau_{ij}^a \int \mathrm{d}y \left[\zeta_I^+(y)\zeta_J^-(y) \mp \zeta_I^-(y)\zeta_J^+\right]\partial_y f^{\pm\alpha}(y)$$
$$= -\mathrm{i}(m_I \mp m_J)\mathcal{T}_{IJ}^{\pm\alpha} + \mathrm{i}\tau_{ij}^a \left[\left[\zeta_I^+\zeta_J^- \mp \zeta_I^-\zeta_J^+\right] f^{\pm\alpha}\right]_\ell \quad (34)$$

Provided the boundary term vanishes, this is indeed the condition (17a) translated to the vector and axial vector couplings $\mathcal{T}^\pm = \frac{1}{2}(\mathcal{T}_R \pm \mathcal{T}_L)$ used in this section.

For the broken gauge bosons A^- the boundary term vanishes since the wavefunctions of the gauge bosons are zero on the boundary. For the unbroken gauge bosons A^+ the vanishing of the boundary term is ensured by the BCs (27):

$$\left[\zeta_I^+\zeta_J^- - \zeta_I^-\zeta_J^+\right]_{y=\ell} = 0 \quad (35)$$

The same conclusion is reached in the orbifold approach, where we impose ordinary orbifold BCs but have to use the singular equations of motion (31). In the coupling to A^+ (34) the resulting additional terms at the boundary cancel, because they are the same for both ζ^+ and ζ^-.

Evidently, the vanishing of the boundary terms requires that the BCs for fermions are consistent with the breaking of the gauge symmetry. For an unbroken left-right gauge symmetry on the brane, the wavefunctions of the A^- are nonvanishing. Thus, in spite of consistency with the variational principle, the BCs (27) violate the unitarity SRs in this case. This is of course expected since the BCs correspond to brane mass terms inconsistent with an unbroken left-right symmetry, but this incompatibility is not apparent in the BC description. Similarly, the cancellation of the boundary terms in (34) demands that all components of the isospin doublets Ψ_L and Ψ_R must satisfy the same BCs (27) so only a mass term consistent with the unbroken $SU(2)_{L+R}$ is allowed.

C. Brane localized kinetic terms and mixing with brane fermions

As discussed in section II, mixing with brane fermions or brane kinetic terms can be used to obtain a mass splitting among the components of the fermion isospin multiplets [15]. We now discuss the consistency of this setup. Brane kinetic terms will be discussed first, the similar case of mixing with brane fermions is discussed below. In the example of Higgsless EWSB, the BKTs are added for the Ψ_R fermions on the brane where a symmetry $SU(2)_L \times U(1)$ is unbroken. To be consistent with the reduced gauge symmetry on the brane, we have to add a BKT with a covariant derivative

$$\mathscr{L}_{BKT} = \kappa \mathrm{i}\bar\eta_{R,i}(\partial\!\!\!/\delta_{i,j} + \tau_{ij}^a A\!\!\!/^a)\eta_{R,j}\,\delta(y) \quad (36)$$

where the A^a are the unbroken gauge bosons only. In the example of Higgsless EWSB, these are the $U(1)$ gauge bosons B. In a more general situation, we can consider a general bulk symmetry group G that is broken to a subgroup H at the boundary by orbifolding

or by Dirichlet BCs. The representation of the fermions under G can then be decomposed into representations of H and we can allow different BKTs on the brane for fermions in the different representations of H.

Taking the BKT (36) into account, the equations of motion for the KK modes become

$$\begin{aligned} \partial_5 \zeta_I^- + m_I \left(1 + \kappa \delta(y)\right) \zeta_I^+ &= 0 \\ -\partial_5 \zeta_I^+ + m_I \zeta_I^- &= 0 \end{aligned} \tag{37}$$

The determination of the BCs and the mass spectrum is reviewed in appendix B 2. The BC corresponding to the BKT is given by (28) and translates to the BC for the KK wavefunctions

$$\zeta_I^-(y)|_{y=0^+} = \kappa m_I \zeta_I^+(y)|_{y=0^+} \tag{38}$$

Again we verify the consistency of this BC with the WIs using the relation (17a). The presence of the BKT modifies the coupling of the fermions to the unbroken gauge bosons to

$$\mathcal{T}^\alpha_{RIJ} = \tau^a_{ij} \int \mathrm{d}y \, \zeta_I^+(y) \zeta_J^+(y) f^\alpha(y) (1 + \kappa \delta(y)) \tag{39}$$

Similarly to (18) we find after integrating by parts:

$$\begin{aligned} m_\alpha g^\alpha_{LIJ} &= -\mathrm{i} \int \mathrm{d}y \, \partial_y \big(\zeta_I^+(y) \zeta_J^-(y)\big) f^\alpha(y) + \mathrm{i} \tau^a_{ij} [\zeta_I^+ \zeta_J^- f^\alpha]_{y_f} \\ &= -\mathrm{i}(m_I \mathcal{T}^\alpha_{LIJ} - m_J \mathcal{T}^\alpha_{RIJ}) \end{aligned} \tag{40}$$

Here the modified coupling constants (39) appear for the KK-modes of the unbroken gauge bosons in the last expression. This follows in the interval approach using the BC (38) and the continuous equations of motion. In the orbifold-approach it results from the discontinuous equation of motion (37) and trivial BCs. For the coupling to the broken gauge bosons, the boundary terms vanish since the gauge boson wavefunctions vanish on the boundary.

We therefore have shown that in the presence of unbroken gauge symmetries on a brane, the modification of the BCs (38) necessitates the modification of the couplings of the fermions to the unbroken gauge bosons according to (39). In the orbifold approach, this modification appears naturally by using covariant derivatives in the BKTs.

Finally, we turn to mixing with brane fermions that is very similar to the case of BKTs [15]. We consider the mixing of the right handed component of the five dimensional fermion $\Psi_R = (\chi_R, \eta_R)$ with a brane localized fermion $\psi = (\chi, \eta)$ at $y = 0$ via a mass term:

$$\mathscr{L}_{Mix} = \delta(y) \left[\mathrm{i} \bar{\psi}_i (\partial\!\!\!/ \delta_{ij} + \tau^a_{ij} A\!\!\!/^a) \psi_j - \mu \bar{\psi}_i \psi_i - M \ell^{\frac{1}{2}} \left(\eta^\dagger_{i,R} \chi_i + \chi^\dagger_i \eta_{R,i} \right) \right] \tag{41}$$

Similarly to the BKTs, only a coupling to the unbroken gauge bosons is present. As discussed in appendix B 3, the solution of the equation of motion of the brane fermions takes the form

$$\psi(x) = \sum_n \beta_n \begin{pmatrix} \frac{m_n}{\mu} \chi_n(x) \\ \eta_n(x) \end{pmatrix} \tag{42}$$

with

$$\beta_n = \ell^{\frac{1}{2}} \frac{M \mu}{(m_n^2 - \mu^2)} \zeta^+_{R,n}|_{y=0} \tag{43}$$

Here the same spinors χ and η appear as in the KK decomposition of the bulk fermions (7). The equations of motion and BCs for the bulk fermions are the same as in the case of BKTs (38) with the replacement

$$\kappa \to \tilde{\kappa}_I = \frac{\ell M^2}{\mu^2 - m_I^2} \tag{44}$$

Because the decomposition of the brane fermions (43) involves the KK-wavefunctions at the location of the brane, the couplings of the fermion KK-modes to the unbroken gauge bosons gets modified:

$$\begin{aligned} \mathcal{T}^\alpha_{RIJ} &= \tau^a_{ij} \int \mathrm{d}y\, f^\alpha(y) \zeta^+_I(y) \zeta^+_J(y) \left(1 + \delta(y) \tilde{\kappa}_I \tilde{\kappa}_J \frac{\mu^2}{\ell M^2}\right) \\ \mathcal{T}^\alpha_{LIJ} &= \tau^a_{ij} \int \mathrm{d}y\, f^\alpha(y) \left(\zeta^-_I(y) \zeta^-_J(y) + \delta(y) \zeta^+_I(y) \zeta^+_J(y) \tilde{\kappa}_I \tilde{\kappa}_J \frac{m_I m_J}{\ell M^2}\right) \end{aligned} \tag{45}$$

Performing the by now usual manipulations to verify (17a), we find again that the singular terms in the equations of motion (or the nontrivial BCs in the interval approach) contribute just the terms needed to compensate for the changed gauge boson couplings:

$$\begin{aligned} -\mathrm{i}(m_I \mathcal{T}^\alpha_{LIJ} - m_J \mathcal{T}^\alpha_{RIJ}) &= -\mathrm{i} \int \mathrm{d}y\, f^\alpha \left[m_I \zeta^-_I \zeta^-_J - m_J \zeta^+_I \zeta^+_J \left(1 + \delta(y) \frac{\tilde{\kappa}_I \tilde{\kappa}_J}{\ell M^2} (\mu^2 - m_I^2)\right)\right] \\ &= -\mathrm{i} \int \mathrm{d}y\, f^\alpha \left[m_I \zeta^-_I \zeta^-_J - m_J \zeta^+_{R,I} \zeta^+_J \left(1 + \tilde{\kappa}_J \delta(y)\right)\right] = m_\alpha g^\alpha_{LIJ} \end{aligned} \tag{46}$$

Considering the limit of sending μ and M to infinity while keeping $\frac{M}{\mu} \equiv \sqrt{\frac{\kappa}{\ell}}$ fixed [15] we have $\tilde{\kappa}_n \to \kappa$ and recover the same BCs and equations of motion as for BKTs. As required by this analogy, the modification in $\mathcal{T}_L$ in (45) vanishes in this limit. Again we have found that modified BCs are consistent with the gauge symmetry only if they correspond to a boundary term invariant under the reduced gauge symmetry and the peculiar form of the additional terms in (45) enforced by gauge symmetry has played an essential role in verifying the consistency.

V. SUMMARY AND CONCLUSIONS

We have applied the sum rules obtained from tree unitarity [12] and Ward identities [6] to discuss the consistency of higher dimensional mechanisms for fermion masses without Higgs bosons. In section III we have introduced an orbifold mechanism to obtain Dirac masses for five dimensional fermions in a chiral theory and have checked the SRs for general orbifold gauge theories involving bulk fermions. Similar to a pure KK gauge theory, the unitarity sum rules are satisfied by interlacing cancellations among the KK-states of both bulk fermions and gauge bosons.

To obtain a mass splitting among the components of the isospin doublets in the SM, the orbifold mechanism is not sufficient and generalized boundary conditions corresponding to brane localized mass and kinetic terms and mixing with brane fermions are employed in models of Higgsless EWSB. In section IV the consistency of these boundary conditions has been discussed. While the approach of [15] to impose BCs consistent with the variational principle at the boundaries of an interval avoids ambiguities from discontinuous

wavefunctions, the consistency of the BCs with gauge symmetries is more transparent in the equivalent description in terms of brane localized terms. We have found that indeed only boundary conditions corresponding to brane localized mass and kinetic terms respecting the reduced gauge symmetry on the brane are consistent with unitarity and WIs so they can be considered as soft symmetry breaking.

Apart from the models of Higgsless EWSB that have served as example in this work, the consistency of the picture of explicit but soft symmetry breaking by brane localized terms invariant under a reduced gauge symmetry is also expected to be important for Higgsless models of gauge unification in higher dimensions.

Another interesting question that is left for future work is the possible description of brane localized mass terms in gauge theories in terms of Scherk Schwarz breaking [17].

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APPENDIX A: EFFECTIVE LAGRANGIANS FOR KALUZA-KLEIN MODES

1. Kaluza-Klein decomposition of gauge bosons

In this appendix we set up our notation for the KK lagrangian and decomposition of the gauge bosons, following the general higher dimensional case discussed in [6].

The 5 dimensional Yang Mills lagrangian is given by

$$\mathscr{L}_5 = -\frac{1}{4} F^a_{AB}(x,y) F^{a,AB}(x,y) \tag{A.1}$$

with the field strength

$$F^a_{AB}(x,y) = \partial_A A^a_B(x,y) - \partial_B A^a_A(x,y) + f^{abc} A^b_A(x,y) A^c_B(x,y) \tag{A.2}$$

Here we include the higher dimensional gauge coupling g_5 in the structure constants. We use a ‘mostly minus’ metric $g_{AB} = \mathrm{diag}(\eta_{\mu\nu}, -1)$. The KK decomposition of the gauge fields is introduced as

$$A^a_A(x,y) = \begin{pmatrix} A^a_\mu(x,y) \\ -\phi^a(x,y) \end{pmatrix} = \sum_n \begin{pmatrix} f^a_n(y) A^a_{n,\mu}(x) \\ -g^a_n(y) \phi^a_n(x) \end{pmatrix} \tag{A.3}$$

The sign of the scalar component is chosen because of compatibility with our conventions for the WIs used in [6]. The wavefunctions $\rho = f, g$ satisfy the differential equation

$$\partial_y^2 \rho^a_n(y) = -{m^a_n}^2 \rho^a_n \tag{A.4}$$

They are taken as orthonormal and satisfying a completeness relation:

$$\int \mathrm{d}^N y \, \rho^a_n(y) \rho^a_m(y) = \delta_{n,m} \tag{A.5}$$

$$\sum_n \rho_n^a(x)\rho_n^a(y) = \delta(y-x) \tag{A.6}$$

In these equations the group indices are not summed over. To diagonalize the interaction of the vector and scalar components we choose [6]

$$\partial_y f_n = m_n g_n \quad , \quad \partial_y g_n = -m_n f_n \tag{A.7}$$

This relations can be used to obtain the effective four dimensional Lagrangian of the KK modes. To simplify our notation, we introduce a multi-index notation with $(a,i) \equiv \alpha$. The cubic interaction terms relevant for the SRs discussed in section III are found as

$$\mathscr{L}^{KK}_{\text{cubic}} = -g^{\alpha\beta\gamma}\partial_\mu A^\alpha_\nu A^{\beta,\mu}A^{\gamma,\nu} - \frac{1}{2}T^\alpha_{\beta\gamma}\, A^{\alpha,\mu}\phi^\beta \overleftrightarrow{\partial_\mu}\phi^\gamma + \ldots \tag{A.8}$$

where the coupling constants are given by

$$g^{\alpha\beta\gamma} = f^{abc}\int \mathrm{d}^N y\, f^\alpha(y)f^\beta(y)f^\gamma(y) \tag{A.9a}$$

$$T^\alpha_{\beta\gamma} = f^{abc}\int \mathrm{d}^N y\, f^\alpha(y)g^\beta(y)g^\gamma(y) \tag{A.9b}$$

The complete lagrangian and the coupling constants in an arbitrary number of dimensions have been given in [6].

2. KK-decomposition for Fermions

We now introduce our notation for fermions on a 5-dimensional orbifold [21, 22]. The Lagrangian is taken as[3]:

$$\mathscr{L}_f = \bar{\Psi}_i(x,y)(\mathrm{i}\partial_M\Gamma^M - m_i(y))\Psi_i(x,y) + \bar{\Psi}_i(x,y)\tau^a_{ij}\Gamma^M A^a_M(x,y)\Psi_j(x,y) \tag{A.10}$$

with the 5-dimensional gamma-matrices

$$\Gamma^\mu = \gamma^\mu\,,\ \Gamma^5 = \mathrm{i}\gamma^5 \tag{A.11}$$

and where the τ^a are the generators of the gauge group in the representation of the fermions. The mass function must be odd under the orbifold transformations (5).

The resulting equation of motion for the free fermion fields is

$$(\mathrm{i}\not{\partial} - \gamma^5\partial_5 - m_i(y))\Psi_i(x,y) = 0 \tag{A.12}$$

We introduce the KK decomposition for the left-and righthanded components:

$$\Psi(x,y) = \begin{pmatrix}\chi(x,y)\\ \eta(x,y)\end{pmatrix} = \sum_n \begin{pmatrix}\chi_n(x)\zeta^-_n(y)\\ \eta_n(x)\zeta^+_n(y)\end{pmatrix} \tag{A.13}$$

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[3] We use a 4-component notation for the spinors with the notation of [26] i.e. $\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu\\ \bar\sigma^\mu & 0\end{pmatrix}$ and $\gamma^5 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$

where η_n, χ_n are 4-dimensional right- and lefthanded spinors that satisfy the appropriate Dirac equations:

$$\mathrm{i}\sigma^\mu\partial_\mu\eta_n - m_n\chi_n = 0 \quad , \quad \mathrm{i}\bar{\sigma}^\mu\partial_\mu\chi_n - m_n\eta_n = 0 \tag{A.14}$$

The KK wavefunctions satisfy the equations of motion

$$(\mp\partial_y - m(y))\zeta_n^\pm = -m_n\zeta_n^\mp \tag{A.15}$$

and completeness and orthogonality relations similar to (A.5) hold.

Inserting the KK decompositions into the Lagrangian and integrating over the fifth dimension results in the interaction lagrangian:

$$\mathscr{L}_{fKK} = \bar{\psi}_I \not{A}_\alpha(\mathcal{T}^\alpha_{LIJ}(\tfrac{1-\gamma^5}{2}) + \mathcal{T}^\alpha_{RIJ}(\tfrac{1+\gamma^5}{2})\psi_J + \bar{\psi}_I\phi_\alpha(g^\alpha_{LIJ}(\tfrac{1-\gamma^5}{2}) + g^\alpha_{RIJ}(\tfrac{1+\gamma^5}{2}))\psi_J \tag{A.16}$$

where we have used a multi-index notation with $(n,i) \equiv I$ and defined the 4 dimensional Dirac spinors $\psi_I = (\chi_I, \eta_I)$. Since only the left or righthanded component possesses a zero mode, either $\chi_{0,i}$ or $\eta_{0,i}$ vanishes, we take this as understood.

The coupling constants are given by

$$\mathcal{T}^\alpha_{R/LIJ} = \tau^a_{ij}\int \mathrm{d}y\,\zeta^\pm_I(y)\zeta^\pm_J(y)f^\alpha(y) \tag{A.17a}$$

$$g^\alpha_{L/RIJ} = \pm\mathrm{i}\tau^a_{ij}\int \mathrm{d}y\,\zeta^\pm_I(y)\zeta^\mp_J(y)g^\alpha(y) \tag{A.17b}$$

APPENDIX B: BOUNDARY CONDITIONS FOR BRANE LOCALIZED TERMS

1. Brane mass terms

The equations of motion obtained in the presence of the brane mass term (3) are given by

$$\begin{aligned} &\mathrm{i}\sigma^\mu\partial_\mu\eta_L + \partial_5\chi_L - \delta(y-\ell)lM_D\chi_R = 0 \\ &\mathrm{i}\bar{\sigma}^\mu\partial_\mu\chi_L - \partial_5\eta_L - \delta(y-\ell)lM_D\eta_R = 0 \end{aligned} \tag{B.1}$$

and the same equations with left and right handed fermions exchanged. The solutions have been found in [19] in the context of Majorana brane masses. Here we discuss only the BCs and the mass spectrum, following [18]. Using the definition of the wavefunctions (30) and the equations of motion (31), we obtain a decoupled differential equation for the ratio of the two wavefunctions $t_m(y) = \frac{\zeta^+}{\zeta^-}$:

$$\partial_5 t_n(y) = (1+t_n^2)m_n - (1-t_n^2)\delta(y-\ell)M_D\ell \tag{B.2}$$

This equation can used to determine the BCs of the wavefunctions, e.g. by integrating over a symmetric interval $[\ell-\epsilon, \ell+\epsilon]$ in the orbifold approach. In the interval approach on may integrate over $[\ell-\epsilon,\ell]$ and define $\int_0^\ell \delta(y-\ell) = \frac{1}{2}$ [18]. Here we follow [15] and displace the delta function to $y = \ell - \frac{\epsilon}{2}$ and impose the same BCs at both boundaries:

$$\partial_y\zeta_n^-(y)|_{y=0,\ell} = 0 \quad , \quad \zeta_n^+(y)|_{y=0,\ell} = 0 \tag{B.3}$$

Integrating over the interval $[\ell-\epsilon,\ell]$ results in the BCs

$$t_n(y)|_{y=\ell^-} = \mathrm{artanh} M_D\ell \tag{B.4}$$

i.e. (27) with $\alpha = \text{artanh} M_D \ell$. Introducing an ansatz compatible with the BCs (B.3) at $y = \ell$

$$t_n(y) = \tan(m_n(y-\ell) - \varphi_\ell(y)) \tag{B.5}$$

the differential equation (B.2) reduces to:

$$\partial_y \varphi_\ell(y) = \frac{(1-t_n^2)}{(1+t_n^2)} \delta(y-\ell) M_D \ell \tag{B.6}$$

From this, we obtain in agreement with [19]:

$$\varphi_\ell(y) = \arctan(\tanh \delta_\ell \epsilon(y-\ell)) \tag{B.7}$$

where $\delta_\ell = \frac{M\ell}{2}$ and $\epsilon(y)$ is the sign function with periodicity 2π

$$\epsilon(y) = \begin{cases} -1, & -\pi R \leq y < 0 \\ 0, & y = 0 \\ 1, & 0 < y \leq \pi R \end{cases} \tag{B.8}$$

While the BCs at $y = \ell$ are satisfied by construction, the BCs at $y = 0$ yield the mass quantization condition

$$m_n = \frac{n}{R} - \varphi_\ell(0) = \frac{n}{R} + \arctan(\tanh \delta_\ell)) \tag{B.9}$$

2. Brane kinetic terms

The equations of motion in the presence of boundary kinetic terms are given by

$$\begin{aligned} \mathrm{i}\,(1+\kappa\delta(y))\,\sigma^\mu \partial_\mu \eta + \partial_5 \chi = 0 \\ \mathrm{i}\bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \eta = 0 \end{aligned} \tag{B.10}$$

To solve these equations, we will impose the BCs

$$0 = \chi(y)|_{y=0,\ell} \qquad 0 = \partial_y \eta(y)|_{y=0,\ell} \tag{B.11}$$

and locate the brane an infinitesimal distance away at $y = \frac{\epsilon}{2}$. The solution for the KK wavefunctions can be found in [23]. Here we follow the same approach as in the case of brane masses to determine the mass spectrum. The equation for t_n is given by

$$\partial_5 t_n = m_n(1+t_n^2) + \kappa m_n t_n^2 \delta(y) \tag{B.12}$$

From this we determine the BC at $y = 0^+$ as

$$t_n(y)|_{y=0^+} = -\frac{1}{\kappa m_n} \tag{B.13}$$

in agreement with (28). In this case, an ansatz compatible with the BCs is given by

$$t_n(y) = -\cot(m_n y - \varphi_0(y)) \tag{B.14}$$

and the resulting equation for φ_0 reads

$$\partial_y \varphi_0(y) = \frac{t_n^2}{(1+t_n^2)} \delta(y) \kappa m_n \tag{B.15}$$

We obtain

$$\varphi_0(y) = \arctan(-\tfrac{1}{2}\kappa m_n \epsilon(y)) \tag{B.16}$$

The BCs at $y = \ell$ result in the mass quantization condition

$$\tan m_n \ell = -\frac{\kappa m_n}{2} \tag{B.17}$$

also found from the explicit solution for the wavefunctions [23].

3. Mixing with brane fermions

In the presence of mixing of brane and bulk fermions [15], the equations of motion resulting from (41) read

$$\begin{aligned} \mathrm{i}\sigma^\mu \partial_\mu \eta_R + \partial_5 \chi_R - \delta(y) \ell^{\frac{1}{2}} M \chi = 0 \\ \mathrm{i}\bar{\sigma}^\mu \partial_\mu \chi_R - \partial_5 \eta_R = 0 \\ \mathrm{i}\sigma^\mu \partial_\mu \eta - \mu \chi = 0 \\ \mathrm{i}\bar{\sigma}^\mu \partial_\mu \chi - \mu \eta - \ell^{\frac{1}{2}} M \eta_R|_{y=0} = 0 \end{aligned} \tag{B.18}$$

The first equation implies the BC

$$\chi_R(y)|_{y=0^+} = \ell^{\frac{1}{2}} M \chi \tag{B.19}$$

We decompose the brane fermions as

$$\psi(x) = \sum_n \begin{pmatrix} \alpha_n \chi_n(x) \\ \beta_n \eta_n(x) \end{pmatrix} \tag{B.20}$$

The coefficients α_n and β_n are fixed by the last two equations of (B.18):

$$\alpha_n = \frac{m_n}{\mu} \beta_n \qquad , \qquad \beta_n = \ell^{\frac{1}{2}} \frac{M\mu}{(m_n^2 - \mu^2)} \zeta^+_{R,n}|_{y=0} \tag{B.21}$$

Using the usual KK-decomposition (7), we then obtain the equations of motion for the KK-modes

$$\begin{aligned} \partial_5 \zeta^-_{R,n} &= -m_n \left(1 + \tilde{\kappa}_n \delta(y)\right) \zeta^+_{R,n} \\ \partial_5 \zeta^+_{R,n} &= m_n \zeta^-_{R,n} \end{aligned} \tag{B.22}$$

with

$$\tilde{\kappa}_n = \frac{\ell M^2}{\mu^2 - m_n^2} \tag{B.23}$$

Thus the wavefunctions satisfy the same equations of motion (37) and hence also the same BCs (38) as in the case of BKTs with the replacement $\kappa \to \tilde{\kappa}_n$.

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